$H^{\pm}W^{\mp}Z$ contribution to the static quantities of the W boson in the context of Higgs-triplet theories

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We calculate the one–loop contribution from the $H^{\pm}W^{\mp}Z$ coupling to the static electromagnetic properties of the W boson. Although this coupling is absent at the tree–level in all Higgs–doublet models, it can be induced at this order in models including Higgs–triplet representations. It is found that the $H^{\pm}W^{\mp}Z$ contribution can be as important as those arising from other couplings including Higgs bosons, such as the standard model coupling $W^{\pm}W^{\mp}H$ or the two–Higgs–doublet model couplings $H^{\pm}W^{\mp}\phi^0$ and $W^{\pm}W^{\mp}\phi^0$, with $\phi^0 = h$, H and A.

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I. INTRODUCTION

The trilinear gauge boson couplings $W^{\pm}W^{\mp}\gamma$ and $W^{\pm}W^{\mp}Z$ are representative of the nonabelian structure of the standard model (SM). It is thus interesting to study any anomalous (one–loop) contribution to them as it is important to test the quantization procedures used for these nonabelian gauge systems. In this context, the CP–even static electromagnetic properties of the W boson, which are parametrized by two form factors, $\Delta\kappa$ and ΔQ, have been the subject of considerable interest in the literature as is expected that future particle colliders be sensitive to this class of effects [1]. The SM contributions to $\Delta\kappa$ and ΔQ were calculated in Ref. [2] for massless fermions, and the top quark contribution was presented in [3]. The sensitivity of these quantities to new physics effects has also been analyzed in some SM extensions, such as the two–Higgs–doublet model (THDM) [4], supersymmetric theories [5], left–right symmetric models [6], $SU(3) \times SU(3) \times U(1)$ models[7], and models with composite particles [8] and an extra gauge boson Z' [9]. $\Delta\kappa$ and ΔQ have also been parametrized in a model independent manner by using the effective Lagrangian technique [10]. In this work we present the calculation of the contribution from the $H^{\pm}W^{\mp}Z$ coupling to the static quantities of the W boson. Charged Higgs bosons appear in many extensions of the SM, such as the popular THDM. In the search for a charged Higgs boson, the $H^{\pm}W^{\mp}Z$ coupling might play an important role, although it is expected to be very suppressed in a Higgs–doublet model. In fact, although the $H^{\pm}W^{\mp}Z$ coupling can have a renormalizable structure, it can only be generated at the one-loop level in multi–Higgs–doublet models [11]. Nevertheless, it can be induced at the tree–level in theories with Higgs triplets or higher representations, though it could be severely constrained by the ρ parameter. This is true in some Higgs–triplet models which do not respect the $SU(2)$ custodial symmetry [12, 13]. It is possible however to construct a model including Higgs triplets that does respect such a custodial symmetry [14, 15, 16], thereby relaxing the constraints from the ρ parameter. The phenomenology of the $H^{\pm}W^{\mp}Z$ coupling has been investigated in the context of the CERN e^+e^- LEP–II collider, the next linear collider [17], and hadronic colliders [18]. Such studies focus mainly on discriminating a charged Higgs bosons arising from a model with Higgs triplets from that induced by a Higgs–doublet model. The main goal of this work is to study the impact of the $H^{\mp}W^{\mp}Z$ vertex on the static electromagnetic properties of the W boson. We will show that the respective contributions to $\Delta\kappa$ and ΔQ may be of the same order of magnitude as those arising from other renormalizable theories which include neutral o charged Higgs bosons, such as the SM or the THDM.

The rest of the paper is organized as follows. In Sec. II we present the calculation of the $\Delta\kappa$ and ΔQ form factors. Sec. III is devoted to discuss the numerical results. Finally, the conclusions are presented in Sec. IV.

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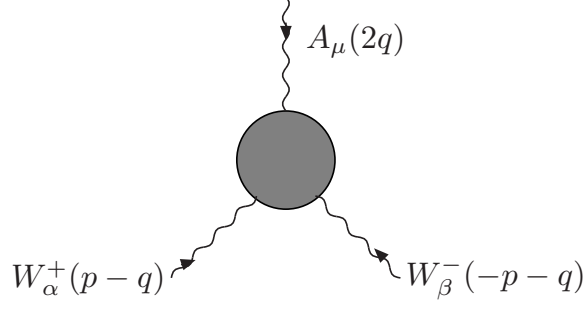†E-mail:jtoscano@fcfm.buap.mx

FIG. 1: Trilinear $W^\pm W^\mp \gamma$ coupling. The loop denotes contributions from charged particles.

II. $H^\pm W^\mp Z$ CONTRIBUTION TO THE ON–SHELL $W^\pm W^\mp \gamma$ VERTEX

Using the momenta depicted in Fig. 1, the most general CP–even Lorentz structure for the on–shell $W^\pm W^\mp \gamma$ vertex can be written as

$$\Gamma_{\alpha\beta\mu} = ie \left(A \left(2p_\mu g_{\alpha\beta} + 4(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu})\right) + 2\Delta\kappa(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) + \frac{4\Delta Q}{m_W^2} p_\mu q_\alpha q_\beta \right). \tag{1}$$

In the SM, both $\Delta\kappa$ and ΔQ vanish at the tree–level, whereas the one–loop corrections are of the order of α/π. These parameters define the magnetic dipole moment μ_W and the electric quadrupole moment Q_W of the W boson:

$$\mu_W = \frac{e}{2m_W}(2 + \Delta\kappa), \tag{2}$$

$$Q_W = -\frac{e}{m_W^2}(1 + \Delta\kappa + \Delta Q). \tag{3}$$

It is worth discussing the origin of the $H^\pm W^\mp Z$ vertex. It has the following renormalizable structure which is dictated by Lorentz covariance

$$i\, s_H\, g\, m_Z\, g_{\mu\nu}, \tag{4}$$

where s_H is a model–dependent quantity which may be very suppressed in models containing only Higgs doublets [11]. As pointed out before, in models with a scalar sector including doublets and triplets, s_H may be considerably enhanced since the $H^\pm W^\mp Z$ vertex is induced at the tree–level. However, not any scalar sector including triplets or higher representations is viable due to the fact that large deviations from the tree–level relation $\rho = 1$ may arise. One possibility is to invoke a tree–level custodial $SU(2)$ symmetry respected by the Higgs sector, which guarantees that $\rho = 1$ at the tree–level. In this case, the existence of a tree–level–induced $H^\pm W^\mp Z$ vertex with strength s_H of the order of unity is possible. Several models of this class have been proposed in the literature, but we will focus on that introduced by Georgi *et. al.* [14], and later considered in [15] and [16]. The Higgs sector of such a model consists of a complex doublet with hypercharge $Y = 1$, a real triplet with $Y = 0$, and a complex triplet with $Y = 2$ [14, 16]. In this model s_H is the sine of a doublet–triplet mixing angle and is given by

$$s_H = \sqrt{\frac{8w^2}{v^2 + 8w^2}}, \tag{5}$$

where v is the vacuum expectation value (VEV) of the Higgs doublet and w is that of both Higgs triplets [16]. As is evident from the above expression, there are two extreme scenarios which have direct implication on the $H^\pm W^\mp Z$ vertex. One scenario corresponds to the case in which the spontaneous symmetry breaking (SSB) of the electroweak sector is entirely determined by the Higgs doublet, *i.e.*, $v \gg w$, implying that $s_H \sim 0$, which means that the $H^\pm W^\mp Z$ vertex is strongly suppressed. The more promising scenario for having s_H of order $O(1)$ corresponds to the case where $w \gg v$, which means that the SSB of the theory is dictated by the Higgs triplet. This scenario is very appealing and so it will be considered below.

We turn now to present the calculation of the contribution from the $H^\pm W^\mp Z$ vertex to $\Delta\kappa$ and ΔQ. This contribution is given by the Feynman diagram shown in Fig. 2. In the unitary gauge, the respective amplitude can
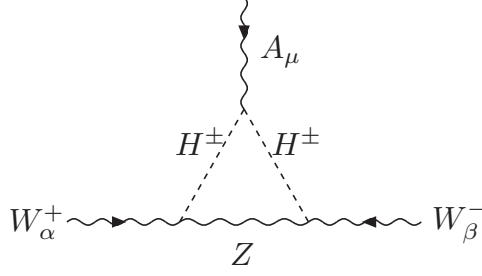
FIG. 2: $H^{\pm}W^{\mp}Z$ contribution to the on-shell $W^{\pm}W^{\mp}\gamma$ vertex.

be written as

$$\Gamma_{\alpha\beta\mu} = -2\,e\,g^2\,m_Z^2\,s_H^2 \int \frac{d^D k}{(2\pi)^D} \frac{(k+p)_\mu (g_{\alpha\beta} - \frac{1}{m_Z^2} k_\alpha k_\beta)}{[k^2 - m_Z^2][(k+p-q)^2 - m_{H^{\pm}}^2][(k+p+q)^2 - m_{H^{\pm}}^2]}. \tag{6}$$

Once the Feynman parameters technique is applied, we obtain the following expressions for the electromagnetic form factors of the W boson

$$\Delta\kappa = 6\,a\,s_H^2 \int_0^1 dx \int_0^{1-x} dy \left((1 - 2x - y) \log \mathcal{R} - \frac{2\,m_Z^2(1 - x - y)}{\mathcal{R}} \right), \tag{7}$$

$$\Delta Q = 12\,a\,s_H^2 \int_0^1 dx \int_0^{1-x} dy \frac{m_W^2(1 - x - y)x\,y}{\mathcal{R}}. \tag{8}$$

with $a = g^2/(96\pi^2)$ and $\mathcal{R} = m_Z^2 - \left(m_W^2 + m_Z^2 - m_{H^{\pm}}^2\right)(x+y) + m_W^2(x+y)^2$. After some calculation one ends up with the explicit solution

$$\Delta\kappa = a\,s_H^2 \left(A_1(x_h, x_z) + A_2(x_h, x_z) \log \left(\frac{x_z}{x_h} \right) + A_3(x_h, x_z) \frac{f(x_h, x_z)}{\delta(x_h, x_z)} \right), \tag{9}$$

$$\Delta Q = a\,s_H^2 \left(B_1(x_h, x_z) + B_2(x_h, x_z) \log \left(\frac{x_z}{x_h} \right) + B_3(x_h, x_z) \frac{f(x_h, x_z)}{\delta(x_h, x_z)} \right), \tag{10}$$

where the A_i and B_i functions are given by

$$A_1(x, y) = \frac{1}{2} \left(1 + 6(x^2 - y^2)^2 + 3(7y^2 - 3x^2) \right), \tag{11}$$

$$A_2(x, y) = 3 \left((x^2 - y^2)^3 - 2(x^2 - y^2)^2 + y^2(x^2 - y^2) + x^2 \right), \tag{12}$$

$$A_3(x, y) = \frac{3}{2} \left((x^2 - y^2)^4 - 3(x^2 - y^2)^3 - y^2(x^2 - y^2)^2 + 3(x^4 - y^4) - x^2(1 + 4y^2) \right), \tag{13}$$

$$B_1(x, y) = -\frac{2}{3} - 2(x^2 - y^2)^2 + 3x^2 - y^2, \tag{14}$$

$$B_2(x, y) = -2 \left((x^2 - y^2)^3 - 2x^2(x^2 - y^2) + x^2 \right), \tag{15}$$

$$B_3(x, y) = -(x^2 - y^2)^4 + 3x^4(x^2 - 1) + y^4(x^2 + y^2) - x^2 y^2(5x^2 - 1) + x^2. \tag{16}$$

In addition, we introduced the definitions $x_h = m_{H^{\pm}}/m_W$ and $x_z = m_Z/m_W$, along with

$$\delta(x, y) = \sqrt{1 - 2(x^2 + y^2) + (x^2 - y^2)^2}, \tag{17}$$

$$f(x, y) = \log \left(\frac{1 - (x^2 + y^2) - \delta(x, y)}{1 - (x^2 + y^2) + \delta(x, y)} \right). \tag{18}$$

In order to cross–check the above results, $\Delta\kappa$ and ΔQ were calculated independently by a general method for reducing tensor form factors. This method, which is an extension of the Passarino–Veltman scheme [19], is described detailed in Ref. [20]. In such a scheme one assumes that $q^2 \neq 0$ and applies the usual Passarino–Veltman reduction.

The reason why one cannot put $q^2 = 0$ in Eq. (6) prior to applying the tensor reduction is that it would require the inversion of a kinematic matrix whose determinant is $\|\mathcal{D}\| = 4\,q^2\left(m_W^2 - q^2\right)$, which evidently vanishes for $q^2 = 0$. Once the form factors for arbitrary q^2 are obtained, the limit $q^2 \to 0$ is taken in order to yield the static quantities of the W boson in terms of two–point Passarino-Veltman scalar functions B_0. Although the limiting procedure $q^2 \to 0$ involves some additional complications since the application of L'Hôpital rule is required, one important advantage of this method is that it can be computer programmed, thereby eliminating the possibility of any mistake. After this scheme is applied, one is left with the following results:

$$
\begin{aligned}
\frac{\Delta\kappa}{a\,s_H^2} &= \frac{1}{2} - \frac{3}{2}\,x_z^2\left(9 + 2\,x_z^2\right) + \frac{3}{2}\,x_h^2\left(1 + 4\,x_h^2 - 2\,x_h^2\right) + \frac{24\,x_z^2}{\delta^2(x_h, x_z)}\left(1 - x_z^2 - x_h^2\right) + \frac{3\,x_h^2}{\delta^2(x_h, x_z)}\left(1 + x_z^2 - x_h^2\right)^3 \Delta B_1 \\
&\quad - 3\,x_z^2\left(x_z^2 - x_h^2 - \frac{4\,x_z^2}{\delta^2(x_h, x_z)}\left(1 - x_z^2 + x_h^2\right)\right)\Delta B_2,
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
\frac{\Delta Q}{a\,s_H^2} &= -\frac{2}{3} + x_z^2\left(3 + 2\,x_z^2\right) - x_h^2\left(1 + 4\,x_z^2 - 2\,x_h^2\right) - \frac{4\,x_z^2}{\delta^2(x_h, x_z)}\left(1 - x_z^2 - x_h^2\right) \\
&\quad - \frac{2\,x_h^2}{\delta^2(x_h, x_z)}\left(x_z^6 + \left(1 - x_h^2\right)^3 - 3\left(1 - x_h^2 + x_z^2\right)x_z^2 x_h^2\right)\Delta B_1 \\
&\quad + \frac{2\,x_z^2}{\delta^2(x_h, x_z)}\left(\left(x_h^2 - x_z^2\right)\left(\left(1 - x_z^2 + x_h^2\right)x_z^2 + x_h^2\left(2 + x_z^2 - x_h^2\right)\right) - x_h^2\right)\Delta B_2.
\end{aligned}
\tag{20}
$$

with $\Delta B_1 = B_0(m_W^2, m_{H^\pm}^2, m_Z^2) - B_0(0, m_{H^\pm}^2, m_{H^\pm}^2)$ and $\Delta B_2 = B_0(m_W^2, m_{H^\pm}^2, m_Z^2) - B_0(0, m_Z^2, m_Z^2)$. These scalar integrals can be solved analytically or numerically evaluated [21]. We compared numerically the latter results with those obtained from Eqs. (9)–(10) and observed a perfect agreement. In this way we make sure that our results are correct.

III. DISCUSSION

The behavior of $\Delta\kappa/(a\,s_h^2)$ and $\Delta Q(a\,s_H^2)$ as a function of x_h is shown in Figs. 3 and 4, respectively. From these figures we can observe that $\Delta\kappa$ is of the order of $a\,s_H^2$ in the range $2\,m_W < m_{H^\pm} < 10\,m_W$, whereas ΔQ is one order of magnitude below. One can also observe that ΔQ decreases more rapidly than $\Delta\kappa$ with increasing $m_{H^\pm}$. It is convenient to compare our results with those arising from other couplings involving Higgs bosons. For instance, the contribution from the SM $W^\pm W^\mp H^0$ vertex was calculated in Ref. [2], in which case $\Delta\kappa \sim a$ and $\Delta Q \sim 10^{-2}a$ for values of m_H in the same range considered for $m_{H^\pm}$ in Figs. 3 and 4. Similar results were found in Ref. [4] for the contributions coming from the THDM couplings $W^\pm W^\mp \phi^0$ and $H^\pm W^\mp \phi^0$, with $\phi^0 = H^0$, h^0, and A^0. We conclude thus that the contribution of the $H^\pm W^\mp Z$ vertex to $\Delta\kappa$ and ΔQ may be as important as those contributions arising from other Higgs boson couplings provided that $s_H \sim 1$, i.e, when the SSB of the electroweak sector is dictated by the Higgs triplets.

We now would like to focus on the decoupling nature of the Higgs boson contributions to the ΔQ and $\Delta\kappa$ form factors. The sensitivity of $\Delta\kappa$ to heavy physics effects as well as the decoupling nature of ΔQ was analyzed in a more general context in Ref. [22]. From Figs. 3 and 4 we can see that ΔQ decouples for a large Higgs scalar mass, whereas $\Delta\kappa$ does not. These results do not contradict the decoupling theorem, which establishes that those Lorentz structures arising from renormalizable operators can be sensitive to nondecoupling effects, whereas those structures coming from nonrenormalizable operators are suppressed by inverse powers of the heavy mass, thereby decoupling in the large mass limit. As far as ΔQ and $\Delta\kappa$ are concerned, the former always decouples for a large mass of a particle running in the loop since its Lorentz structure is generated by a nonrenormalizable dimension–six operator; on the contrary, $\Delta\kappa$ may be sensitive to nondecoupling effects as the Lorentz structure associated with this quantity is induced by a renormalizable dimension–four operator. In this context, the decoupling properties of ΔQ and $\Delta\kappa$ have been discussed in the context of all of the renormalizable theories considered up to now in the literature [2, 7]. For instance, in the SM the heavy Higgs mass limit yields $\Delta Q \to 0$ and $\Delta\kappa \to a$ [2]. As far as the THDM is concerned, $\Delta\kappa \to 2\,a$ and $\Delta Q \to 0$ when m_{ϕ^0} becomes very large and $m_{H^\pm}$ is kept fixed, whereas $\Delta\kappa \to -a$ and $\Delta Q \to 0$ in the opposite scenario [4]. In addition, in the same model, both $\Delta\kappa$ and ΔQ vanish when both m_{ϕ^0} and $m_{H^\pm}$ are very large. In our case, we obtain that $\Delta\kappa \to -(a\,s_H^2)/2$ and $\Delta Q \to 0$ in the heavy charged scalar mass limit. These values follow readily from Eq. (7).

It is interesting to note that our results can also be used to obtain the contribution to the static quantities of the W boson from the $H^\mp W^\pm Z'$ coupling, with Z' the extra neutral gauge boson appearing in theories with an extra $U(1)$.
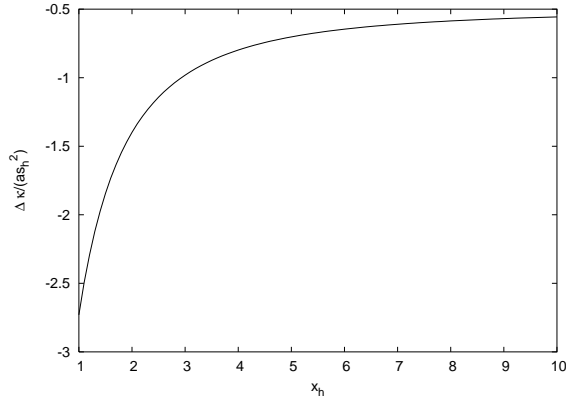
FIG. 3: $\frac{\Delta\kappa}{a\,s_H^2}$ as a function of $x_h = \frac{m_{H^\pm}}{m_W}$.
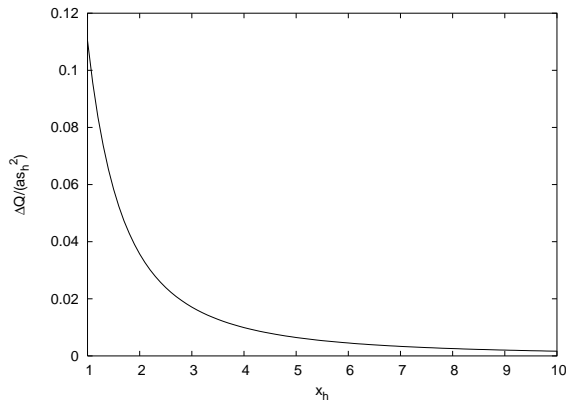

FIG. 4: $\frac{\Delta Q}{a\,s_H^2}$ as a function of $x_h = \frac{m_{H^\pm}}{m_W}$.

This contribution has yet been calculated in the context of an extra $U(1)$ superstring inspired theory [9]. According to these authors, their results were obtained in the Landau gauge. We have compared numerically our results with those presented Eqs. (12) and (13) of Ref. [9] and found no agreement. As pointed out above, our results were cross–checked by making a comparison between the results obtained via the Feynman parameters technique and those obtained by the slightly modified version of the Passarino–Veltman scheme described in Refs. [7, 20]. These two methods are independent and allows us to make sure that our results are correct.

IV. CONCLUSION

The novel feature of a Higgs–triplet representation is the presence of a tree–level $H^\pm W^\mp Z$ vertex, whose strength may be of the order of the unity provided that the SSB of the electroweak sector is dictated by the VEV of the neutral components of the Higgs triplet. This class of models can be viable as long as a tree–level custodial $SU(2)$ symmetry is respected by the Higgs potential. The phenomenology of the $H^\pm W^\mp Z$ coupling is thus be very appealing. Any direct or indirect evidence of the this coupling would be a clear signal of the existence of a scalar sector comprised by Higgs triplets. We have studied the impact of this vertex on the static electromagnetic properties of the W gauge boson. We found that the respective contributions to the form factors $\Delta\kappa$ and ΔQ are as important as those predicted by the SM coupling $W^\pm W^\mp H^0$ or those arising from the THDM couplings $W^\pm W^\mp \phi^0$ and $H^\pm W^\mp \phi^0$.

We would like to point out that our results can also be used to evaluate the contributions from the $H^\pm W^\mp Z'$ vertex to the magnetic moments of the W boson, where Z' is the neutral boson which appears in theories with an extra $U(1)$ gauge symmetry. We note that our results, (9) and (10), disagree with those presented in Ref. [9] for the contribution from the $H^\pm W^\mp Z'$ coupling in an extra $U(1)$ superstring inspired theory. Finally, We emphasize that our results were cross–checked by making a comparison between the results obtained via the Feynman parameters technique and those obtained by the slightly modified version of the Passarino–Veltman scheme described in Ref. [20].

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